



Analysis of a chaotic time series simulated by the density-dependent two age-classes model: $x_t = r \cdot y_t \exp(-0.01 \cdot (y_t + x_t))$, $y_t = 0.2 \cdot x_t \exp(-0.01 \cdot (y_t + x_t)) + 0.8 \cdot y_t \exp(-0.05 \cdot (y_t + 0.5 \cdot x_t))$. a , $r = 112.5$; b , $r = 117$. Small boxes, the corresponding attractors; solid lines, analysis of the full sequence $(x_t)_{0 \leq t \leq 1,000}$; dashed lines, analysis of the sub-sequence $(x_{7i})_{0 \leq i \leq 1,000}$.

sists when attractors are no longer fragmented (this is obtained by tuning one parameter of the model). Now the overall periodicity is far less obvious, but probably responsible again for our unexpected results.

These phenomena are illustrated in the figure. In *a*, the overall period of the strange attractor is 7, and chaos is not detected on 1,000 consecutive points: the correlation coefficient between predictions and actual data points shows no dependence on the prediction-time interval. However, restricting the analysis to a sub-sequence of points located in one of the seven pieces of the attractor provides the signature of chaotic dynamics¹. In *b*, a similar comparison is performed; the conclusions are identical, although the overall periodicity is not evident.

Supported by recent analyses of models for single-species populations (R. H. F. and M. Gatto, in preparation) or multiple-species food chains⁵, there is an increasing belief that chaos may commonly exist, in natural systems, in a form which appears to be regular oscillations over short timescales⁴. Therefore, to improve the applicability of the Sugihara and May method, one should first

test any time series $(X_t)_{t>0}$ for an eventual cyclic trend (with period τ), and then look for chaos in the sub-sequence $(X_{\tau \cdot i})_{i>0}$. (However, this precaution generates additional need for a rather long time series, all the more so as τ increases.) Real data previously interpreted as reflecting noisy cycles should be reconsidered in this way.

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SUGIHARA AND MAY REPLY — One problem in trying to reconstruct the attractor that may underlie an apparently chaotic time-series is that we often have data for only one variable in what is, intrinsically, a multivariate system. Following Takens (F. Takens, *Lect. Notes Math.* **898**, 366–381; 1981), we may 'embed' our one-dimensional time series in an E -dimensional phase space by, in effect, running a fork with E tines, spaced at intervals τ , along the time series. By regarding the E data points the tines touch as a single E -dimensional point, we generate a constellation of points in E -space.

The difficulty is that, in general, we have no *a priori* knowledge about the proper values of E and τ ; that is, about the number of tines and their spacing. Very often, commonsense has resulted in the data being collected at intervals that roughly correspond to the appropriate value of τ , so that the difficulties primarily lie in finding the best E . Mainly for this reason, our paper focused on the embedding dimension E (the number of tines on the fork); although we did refer to the question of appropriate τ -values (spacing between the tines; see our Fig. 3*b*), we gave it little emphasis. Cazelles and Ferrière make the useful point that appropriate spacing — appropriate τ -values — can be important.

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Changes in Earth rotation rate

SIR — Variations of the Earth's rotation rate with periods less than two years are due to variations in the zonal circulation of the upper atmosphere. The longer-period variations, called decade variations, are attributed to exchanges of angular momentum between the Earth's rigid mantle and its liquid core. Ten years ago, we proposed¹ the existence of a significant correlation between these decade fluctuations of the Earth's rotation rate and those of a properly chosen indicator of secular variation of the geomagnetic field. Based on the observation of a secular variation impulse in 1969–70, we predicted² that an acceleration of the rotation rate would take place in the early 1980s. This indeed was what happened³. Observation at that time of more recent secular variation (a new sharp change of slope in 1978) led to a second prediction of a deceleration in rotation rate starting about ten years later². We can now confirm this prediction and the 9 ± 2 -year lag between the two data sets (see figure).

In 1981, we suggested that the correlation was due to electromagnetic coupling between the core and mantle, the source of this coupling lying with the westward bodily drift or rotation of the external layers of the core with respect to the mantle. Much work has been done since then to study the flow at the core–mantle boundary. Westward drift appears now to be limited to the Atlantic hemisphere and has much less significance than previously thought; it is now even doubtful whether any body rotation exists at all⁴.

Electromagnetic coupling is, on the other hand, faced with very significant difficulties in accounting for the observed very rapid slope changes in the Ω curve⁵. The favoured physical mechanism appears to be topographic core–mantle coupling, first proposed by Hide⁶. If the core–mantle boundary displays topography (departures from axial symmetry), flow at the surface of the core exerts a net torque on the mantle, except for very special flow configurations⁷.

If the amplitude of the topography reaches a few hundred metres, as inferred from nutation data, or even larger, as proposed by some seismologists, the calculated torque is found to be orders of magnitude larger than allowed by the rotation data (for example, ref. 8). One must then assume that flow in the upper core remains on average locked in a specific configuration with respect to topography of the core–mantle boundary such that the resulting net torque is far

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Scientific Correspondence

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