Introduction to	Empirical Dynamics
A Quick New Approache	Look Ahead at s to Difficult Problems
Nonlinearity, Prediction	on, Coupling and Causation
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An Introduction to Empirical Dynamics:

An Inductive Data-Driven Approach

My aim here will be to speak to a particular perspective that may be of special relevance as we move away from simple 20th century reductionist toy models based on fundamental principles, toward trying to understand how messy natural systems behave. For example, while we can easily write down an accurate equation for diffusion of gases in a test tube, modeling oxygen concentrations at depth in a large lake (where biology, complex chemistry and physical currents intervene) is impractical with explicit equations. Empirical models, which infer patterns and associations from the data (instead of using hypothesized equations), represent an alternative and highly flexible approach.

All this is being made possible by the era of Big Data. 21st century holistic science is being enabled by a boon in available data, and EDM is a useful approach for data exploration. The math itself is not especially challenging, however the resonance of understanding that can be achieved with a deeper understanding of the implications of simple classical assumptions like equilibrium,

Two key points of emphasis are:

1. Detecting causation to uncover mechanism in natural nonlinear dynamic systems

2. Forecasting as a rigorous way to validate understanding.

Two key points of emphasis for inductive data-science are as follows:

- I. Detecting causation to uncover mechanism in nonlinear dynamic systems
- 2. Forecasting as a rigorous way to validate understanding

Every one gives lipservice to nonlinearity, but few actually acknowledge it or truly understand it.

I would like to look at some pedagogical points that are often swept under the rug... but I suggest they actually redefine how we should study these systems

"Correlation versus Causation"

Two main elements:

I.The fact that nature is dynamic -temporal sequence matters

2.The fact that nature is nonlinear - context/connectivity matters 1) The fact the nature is dynamic - temporal sequence matters

Nature is best understood as a movie rather than a snapshot.

2) The fact that nature is nonlinear

Meaning it consists of interdependent parts...that are nonseparable – context matters

It cant't be studied as independent pieces. Rather each piece needs to be studied in the context surrounding it.

Let's start with an example....

stop



these two variables (eg.**species and a forcing variable**) are uncorrelated. However they are in fact **deterministically coupled**. *click* *click* These time series are produced from a simple coupled logistic difference system. ... an example of nonlinear dynamics.

A = [[3.5 0.1][0.02 3.8]]

So

x1(t+1) = x1(t) * (3.5 - 3.5 * x1(t) - 0.1 * x2(t)) x2(t+1) = x2(t) * (3.8 - 3.8 * x2(t) - 0.02 * x1(t)) x1(0) = 0.2





This is interesting because this blue disc is the realm of biological systems. *stop*

And within this realm...a further consequence of nonlinearity demonstrated in the **model example** was the phenomenon of **mirage correlation**...



Mirage Correlation

a further consequence of nonlinear dynamics

In retrospect, after what I just showed you, this converse property should be wellknown, but apparently it is not. It contradicts a currently held view that correlation is a "necessary" condition for causation.

Tufte is a distinguished statistician and political scientist from Yale

These ephemeral or mirage correlations are "associations that come and go and even switch sign"

This perverse tendency of nonlinear systems is the bane of Ecology and of financial modeling relationships that appear then disappear as soon as you try to exploit them.

Let's see an example...



Here is another example from SoCal. *click* Using data up to 1991, a significant positive relationship was found between sea surface temperature and sardine production (true for two different measurements of productivity (recruitment)). This was reported in 1994 and was subsequently written into the state law for managing harvests.

click *click* However, when data from 1992-2008 are included (17 additional data points), the correlation seemed to disappear (in both cases), causing the plan to be suspended in 2010... where it now stands.

Myers, 1998 A meta-analysis of 74 environment-recruitment (fish productivity) correlations reported in the literature.

• Only 28 out of 74 held to retest when data subsequent to the original study was added.

(Fewer now: sardine-temperature was still successful at that time)

Empirical Dynamics (EDM)

•A holistic approach for studying complex systems from time-series observations

• Involves the study of dynamic attractors

Another famous example from fisheries-

was a meta-analysis on 74 environment-recruitment correlations that were reported in the literature. These correlations were retested using additional data obtained subsequent to the publication of each of the studies – only 28 of the 74 correlations remained.

(Certainly fewer now, since sardine-temp was among the ones that still help up at the time of Myers analysis)

Relationships we thought we understood seemed to disappear. This sort of thing is familiar in finance where relationships are uncovered but often disappear even before we try to exploit them.

(Species included Atlantic cod, Northern Anchovy, Sockeye salmon, Maine Lobster, and many others).

So, how to address this?

The approach I will present here is based on nonlinear state space reconstruction which I refer to here with the less technical name... empirical dynamics. EDM is a holistic data-driven approach for studying complex systems from their attractors. It is designed to address nonlinear issues such as mirage correlation.

I am now going to play a brief video animation that will explain all. (my son made this for me when he was a junior at Columbia). The narration is by Robert May. **click**click**

It is an alternative to the theoretical expedient of constancy and decomposability. The common assumption that natural systems are in equilibrium has legitimized reductionism and the use of linear methods. For example, to study dynamics —we can use local linear stability analysis. -constancy in pairwise interactions- a picture of independence; dynamics are reduced to random motion around a mean. Time (sequence of events) is irrelevant

However, if we don't make this assumption then we need to account for dynamics that exhibit nonlinear state dependence

-nonlinear state dependence -> interdependence

This has important implications for how to study nature (holistically), and for identifying causal drivers and networks.



Time series are observations of motion occurring on an attractor. A time series is an "observation function" of the dynamic mechanism



And depending on when they are viewed, relationships among variables can appear to change... giving rise to mirage correlations

Over the short term there might be correlations, but over the longer term If one were to study this system by plotting randomly sampled values of X and Z there would be no correlation. This problem only becomes coherent when temporal sequence is included.

Let's look at a real example.



Constructing Empirical Dynamic Models: Takens' Theorem

> from "Detecting Causality in Complex Ecosystems" Sugihara et al. *Science* (2012) narration by: Robert M. May

> > © September 2014



To recap, Takens theorem says any one variable contains information about the others. This allows the construction of attractors from a single variable using lags as proxy coordinates.

Constructing attractors from time series data is the basis of the Empirical Dynamic approach. -univariate -mulitvariate -mixed embedding

let's look at some examples...



When time series have no relationship to each other, plotting them together as a trajectory in a multivariate state space yields a tangled mess. There is no sign of structure or pattern— and there shouldn't be!

[[Click to show 2nd attractor]]

In contrast, when interrelated time series are plotted together, the trajectory forms a manifold. Here, we show three time series from Mono Lake, a saline lake in California with a simple food web. The trajectory forms a coherent pattern that we can then study to make predictions and gain insights into the interactions between the variables.



Here we have an ecological example: attractors constructed from time series for sockeye salmon returns for the Fraser River, Canada. ...Again, using time-lagged coordinates.

-Each point represents a 3-year history.

-Basically, the trajectories run along consecutive 3-year histories. The fact that **3 dimensions are sufficient to unfold** the trajectories suggests it may be **possible** to make a reasonable **3-factor multivariate model** with wellchosen mechanistically relevant time series (**eg. river discharge, SST and spawning stock abundance**)

*****Full Stop*****

State Shifts in Nanog Stem cell transitions from the undifferentiated (pluripotent) to the differentiated state		
A comparison of taking a static versus a dynamic view		
Verma Lab, Salk & Sugihara Lab SIO Gerald Pao, Ethan Deye		

Viewing Nanog Gene Expression from a Statistical Snapshot (Static View)



- Nanog gene manipulation
 Mouse stem cells engineered to produce GFP when Nanog gene is actively expressed
- In static snapshot, can see that some cells have high expression, some low.

Collaboration with Verma Lab, Salk Gerald Pao, Ethan Deyle

This is another example.

Nanog is a transcription factor that keeps stem cells pluripotent.

Mouse stem cells are engineered to produce Green Florescent Protein (GFP) when the Nanog gene is actively expressed.

In this snapshot we see that some cells have high expression (green) and some low (dark)....low states are when the cell differentiates. We don't see much in between.

Transition to low states was believed to be stochastic

Nanog works to maintain pluripotence even w/o lif (leukemia inhibitory factor))

GFP was inserted into the nanog locus in one strand only



This idea of cells randomly transitioning between states comes from taking a static statistical view

That is, we assume the probabilistic average you see across this ensemble of cells represents what we expect of any individual cell.

When we do this and plot the histogram we see 2 discontinuous states. (*click*)

and the reigning hypothesis is that switching states is purely random.



In fact we can draw out the attractor (in 3D here, though the 5D is the best embedding dimension for these time series data) and see that the transition between states involves visiting two different parts (lobes) of a manifold.

This suggests that the rapid transitions between states may be understandable as a nonlinear dynamic phenomenon rather than a purely stochastic one.

This is nascent work, but I think it conveys the idea that dynamic tools are useful when studying what is essentially a dynamic process.

And in particular, that understanding causal interactions in such systems really requires a nonlinear dynamic perspective.

Prediction	Two methods: Simplex projection S-maps
Out of sample forecasting is a rigorous way to validate understanding Model fitting is not prediction!	In my view, prediction should be the standard (measure of merit) for validation in science. (Indeed it seems odd that it is not generally so) Fisheries models GCM's Hydrology etc.
	We will present two basic methods:
	Simplex projection and S-maps.
	Many other possibilities exist. These are just two very simple ones.
Forecasting with Empirical Dynamics	
Simplex Projection	
• S-maps	



Simplex Projection

4

This is simply forecasting using nearest neighbor analogues.

Nearest neighbors on the attractor are "points with similar histories"

Let's see how this works!

Again, each "point" on the manifold is a "history vector".... a history fragment



Nearest neighbors are "points with similar histories"



Track where they went



tent.delta.series 0 -x- 1000 -1 -y- 0.5

White noise (statistically not predictable) First half second half let's see what we get with time-lagged coordinates

The prediction is a weighted average of the neighbors fates.



This is what we find in 3 dimensions (Fork with 3 tynes..., X,Y, Z).





How did I choose 3 dimensions to embed this?

How did I choose 3 dimensions to embed this?

---> Predictability

The embedding with the best predictability is the one that best resolves singularities... best unfolds the attractor.

Let me explain.





all points are weighed equally... to produce a single global linear map (fitting a single map through all the points)



as theta is tuned upwards the points close to the current state (the predictee) are weighted more heavily when computing the map. Now we no longer have a single map, but a different map at each point.





Again, S-maps are used to identify curvature in the manifold

Curvature is ubiquitous

Nonlinear Attractors Are Ubiquitous in Nature

Hsieh et. al. Nature Vol 435 May 2005 Table 2 | Analyses of key North Pacific biologi Timescale Weekly Monthly Quarterly Biannual Annual Biological data P-value <0.01 0.134 <0.01 0.164 0.040 0.078 0.322 0.033 0.063 0.273 <0.01 0.213 <0.01 0.118 0.168 0.148</br> Best 0.539 0.542 0.715 0.744 0.677 0.566 0.603 0.502 0.576 0.448 0.656 0.634 0.654 0.634 0.484 0.484 Scripps Pier diaton Scripps Pier diaton 0.139* 0.083 0.031* 0.020* 0.027 0.015 0.060* 0.092 0.017 0.440* 0.117 0.767* 0.772 0.118 0.772 0.168 0.078 0.3 0.05 1.6 0.6 1.4 1.2 0.4 0.2 0.6 0.4 0.3 0.18 0.2 0.7 830 206 3,220 1,400 1,736 1,736 868 805 350 1,190 63 63 63 63 63 63 COFI coastal larval fis I coastal-oceanic larva other examples Albacore (Glaser et al. 2011) ٠ Bluefin Tuna (Fromentin & Powers 2005) ٠ Sheep (Grenfell et al. 1998) Diatoms, Childhood diseases (Sugihara & May 1990) • cardiac rhythms, sunspots, gravitational flux, fruit fly behavior, neurobiology, gene expression etc.

An Example of Nonlinearity: Episodic Fluctuations in Larval Supply

Dixon, Milicich and Sugihara Science (1999)

Example of Nonlinear State Dependence Pomacentrid Larval Supply at Lizard Island

Dynamic state dependence is ubiquitous

This actually has profound implications for how we can study these systems.

Phototropic Damsel fish larvae caught in light traps on the reef



What this embedding result tells us

- Pomacentrid larval supply is a low dimensional nonlinear process. (rho=0.78, n=256)
- The optimal embedding for the pomacentrid data is 3 dimensions.
- Therefore it should be possible to construct a model containing 3 variables that is similar in forecast skill as the univariate lag-coordinate model.

"Leverage with multiple timeseries" Look for a *mechanistic* model by searching parallel time series of key environmental variables.

- Construct mechanistic embedding models for prediction by a trial and error search of parallel physical time series.
- Repeat this linearly to construct the best multivariate ARMA model (AR3).

By multiva	riate simple:	x projection	(nonlinear
search) fo	und that the	e best variat	les were.

- 1) %night time illumination lagged 19days.
- 2) cross shelf wind lagged 1 day (best not lagged, but this represents forward information).
- 3) moderate wind speeds lagged 16-19 days.
- Linear rho= 0.27, nonlinear rho=0.82

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<section-header> Using S-map to track changing interactions in real time PROCEEDINGS B rp.royalodetypublishingar Tracking and forecasting ecosystem interactions in real time Researd Execution Submark Torking And South A

imprintia toosis to track, and pretext them in mutue are lacking: prove we down a second second second second second second second second second methods are second second second second second second second methods with the second second second second second second remonstration of the second second second second second second remonstration second second second second second second remonstration second sec



eg. This paper that appeared last year in PRSB used S-maps to show how species interactions vary in time depending on where on the attractor you are. That is, it showed how to make real time measurements of interactions that are state-dependent.

The basic idea is as follows..

The S-map involves calculating a hyperplane or surface at each point as the system travels along its attractor. This involves calculating the jacobian matrix whose elements are the partial derivatives that measure the effect of the system variables on each other.

Note the the embeddings here are multivariate – In native coordinates (not lags).

Again, the coefficients are fit "sequentially" for each location on the manifold using weighted linear regression, with strongest weight given to nearby points, as shown in

the previous slides.

In a stable equilibrium system these coefficients are fit to a single equilibrium point and are fixed and unchanging. In S-maps, however, the values are state dependent they vary depending on location on the attractor, $x(t^*)$.

Thus, by computing sequential jacobians, the S=map tracks interactions (partial derivatives) that change with the evolving state of the system.

What is really nice about this is that it is easily accomplished with real field data.





Here is an example applied to data from a marine mesocosm. (Huisman)

Note that competition between the two main grazers (shown in red), calanoid copepods and rotifers, waxes and wanes.... competition occurs only occasionally...and very episodically ... why?

Thus, we have now have a practical tool for probing changing interactions.



Granger Causality

If the following is true

 $\sigma^{2}\left\{\left(Y_{2}|\overline{U}\right)\right\} < \sigma^{2}\left\{\left(Y_{2}|\overline{U-Y_{1}}\right)\right\}$

Then Y_1 "Granger Causes" Y_2

U is the universe of all causal variables

Dynamic Causation

- Time series variables are causally related if they are coupled (pertubing one variable perturbs the other) and belong to the same dynamic system.
- This can be tested with cross mapping.

Let's now see how EDM deals with causation

Here we are trying to predict Y2 from U (left side)

If we now remove YI, and predictability declines, YI was causal

The problem, however, is that for dynamic systems.... cannot remove YI (according to Takens information about each variable is encoded in all of the others...

In dynamic systems, time series variables are causally related if they are coupled and belong to the same dynamic system... **read slide**

"Information about the aggressor is found in the victim." as it were

Detecting Causality in Bound of the second s	The basic idea was described in this article, and is summarized in the following video clip last one.
<section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><section-header><text></text></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header></section-header>	Here is another video clip
Convergent Cross Mapping (CCM) • If X causes (influences) Y then, Y contains information about X that can be used to predict (recover) X. • That is, states of X can be recovered from the history of Y.	Convergent cross mapping (CCM) involves recovering states of the causal variable from the the affected variable. If this is possible, then causal influence is established.



A necessary condition is that the cross map estimate should improve (converge) with time series length

L = library size or time series length

Let's see some examples



Example 1: This is the classic pred-prey experiment that Gause made famous. didinium=rotifer predator paramecium=prey

Cross mapping in both directions indicates bi-directional causation. red = effect of pred->prey blue = effect of prey -> pred

Units are in 1/2 days.... lag 2 = 1 day





This is a field example:

Sardines and anchovies show reciprocal abundance patterns in the 20th century suggestive of competition.

With cross mapping, however *point* we see that there is no reciprocal information here. Sardines do not affect anchovies and visa versa. However, for both species we find clear evidence of convergence with SST. That is, the time series for both sardines and anchovies contain information about ocean temperatures.

Red Tides in La Jolla McGowan et. al. Ecology (2017) • Observations of episodic red tides in La Jolla date back to 1900 ... regime-like in occurrence

- Prediction elusive, but are hypothesized to arise during very specific environmental conditions
- No obvious correlations



This was exactly the case with the temperature anomaly we saw earlier

a final ecological example:

Episodic Red Tides around Scripps are a classic example of something that no one has been able to predict. They have been thought to be regime-like, and the mechanism for the rapid transition to this state remained a mystery for over a century.

Despite a half dozen or so Scripps Theses showing in principle (by experiment) that environmental drivers should be important, no obvious field correlations have been found. (between environmental variables and chlorophyll-a).



The **absence of environmental correlations** suggests that the events cannot be described by linear dynamics,

and this is confirmed by an S-map test for nonlinearity and by the **significant predictability** found with nonlinear forecasting using univariate simplex projection.

... these predictions are for events driven by internal deterministic dynamics.



The **absence of environmental correlations** suggests that the events cannot be described by linear dynamics,

and this is confirmed by an S-map test for nonlinearity and by the **significant predictability** found with nonlinear forecasting using univariate simplex projection.

... these predictions are for events driven by internal deterministic dynamics.



However, when we examine just the bloom days (n=169), prediction (univariate simplex) is not nearly as skillful.

This suggests that internal dynamics alone cannot explain red tides, and that to do so we must explicitly account for stochastic external drivers.

Cross-mapping vs. Correlation

	Ei	Lag (wks)	chl-a ⇒ <i>E</i> i	<i>E_i</i> ⇒ chl-a	correlation
Nutrient	nitrate	1	0.24	0.04	-0.12
	phosphate	1	0.03	0.20	0.10
	silicate	2	0.16	<0	0.15
Linear	nitrite	1	0.24	0.01	-0.13
History	N:P	1	0.32	0.15	-0.11
	N*	2	0.02	<0	-0.05
	Si*	2	0.16	<0	0.16
	T _{surface} (SIO)	3	0.19	<0	-0.12
Stratification	T _{bottom} (SIO)	1	0.19	0.08	-0.03
	Ssurface (SIO)	0	0.10	0.14	-0.24
	Sbottom (SIO)	0	0.08	0.15	-0.21
	ρ _{surface} (SIO)	3	0.32	<0	-0.04
	ρ _{bottom} (SIO)	1	0.34	0.05	-0.04
	T _{surface} (buoy)	3	0.26	<0	-0.15
	wind _u (buoy)	2	0.21	0.12	0.09
			alleriner Carl	Augusta .	n=169



The candidate variables fall into two loose categories:

1. variables that summarize nutrient history (CLICK)

2. and variables related to stratification and mixing (CLICK).

Again, (CLICK) if you look with cross correlation, there is very little suggestion of environmental forcing.

However, when you look with CCM (CLICK), you can see most of the suspected candidate variables do in fact show causal influence in the time series data from field observations.

Therefore, including these variables as coordinate axes in multivariate EDM

Story of class...

This is TRUE out of sample forecasting.

Indeed, leave-one-out cross-validation over the entire 30-year (1600 point) time series gives very few false forecasts that a bloom will occur (e.g., for some model ensembles as few as 34 false positives and 19 false negatives).

Thus, we have begun to build a valid understanding of the causal mechanisms and more importantly we can forecast red tides with some accuracy.

Some example studies

Dynamical evidence for causality between galactic cosmic rays and interannual variation in global temperature

- Anastasios A. Tsonis^{a,1}, Ethar and Geli Wang^d
- PNAS
 - Experimental studies suggest that cosmic rays could affect global temperature (via cloud formation).
- CCM can distinguish between short-term dynamics (i.e., cloud formation) and long-term dynamics (i.e., climate change) by examining first-difference temperature vs. raw temperature
- Cosmic rays influence only year-to-year variations in temperature

LETTERS

The increase in CR incidence in the 20th century has been * used to suggest that the observed climate warming is natural and not due to man.

* This study used CCM to examine this potential effect. It found no evidence for CR causing the 20th century warming trend. But it did find an effect on interannual time scales... resonates with experiments the show how CR could affect cloud formation.

nature climate change

Causal feedbacks in climate change

Egbert H. van Nes1*, Marten Scheffer¹, Victor Brovkin², Timothy M. Lenton³, Hao Ye⁴, Ethan Deyle⁴ and George Sugihara4*

- Confirms by direct observation the well-established mechanism that greenhouse gases (CO₂ and CH₄) affect temperature. An immediate effect.
- · Confirms the controversial link of temperature affecting greenhouse gases, producing positive feedbacks. A delayed effect.

This study involved the analysis of the Vostock ice core time series data to see if there is direct observational evidence for causal effects



This one focused on forecasting. It was aimed at providing better production forecasts for Canada's iconic sockeye salmon industry.



2016 winner of William James Prize

Untangling Brain-Wide Dynamics in Consciousness by Cross-Embedding

Satohiro Tajima 🗃, Toru Yanagawa, Naotaka Fujii, Taro Toyoizumi Published: November 19, 2015 • http://dx.doi.org/10.1371/journal.pcbi.1004537

- "The model-free method reveals a consciousness-related hierarchy of cortical areas, where dynamical complexity increases along with crossarea information flow.
- "This approach reveals a universality of inter-areal interactions and complexity in conscious brain dynamics, demonstrating its wide application to deciphering complex neuronal systems."

And this is an application of these methods to understand environmental drivers of flu epidemics. What is interesting here is that we were able to identify AH as causal and find a specific temperature threshold 75F below which higher AH reduces flu transmission and above which it increases flu transmission.

There are many other factors of course, but AH is certainly one of them.

Showed how the approach can be developed to index brain states.

	Now this is getting "preachy"
Closing Remark	Static Theoretical Ideal vs. Dynamic Reality
"There is a fundamental disconnect between the biological interactions that we observe and the common (linear/ reductionist) assumptions of the framework that we use to study them."	Flip book analogy

Summary Statement:

Static Theoretical Ideal vs. Dynamic Reality

Wordy Manifesto Despite the known reality and ubiquity of nonlinear dynamics, and the costs associated with unanticipated threshold phenomena or tipping points, nearly all attempts to understand them in applied contexts (outside of formal studies of turbulence) have used incorrect linear statistical tools (static analytical tools based on a classical linear paradigm). This paradigm based on stable, stationary equilibrium points or cyclic equilibrium dynamics allows systems to be studied piecewise as a decomposable sum of independent parts; a tractable approach that applies robustly in designed engineering contexts. As a consequence, an extensive methodological tool chest has evolved for analyzing linear (separable) systems. Indeed the ubiquity of available tools seems to be the main reason why these methods and concepts continue to be used in non-engineering contexts, despite the obvious problem that they do not match our current views of how most real (nonengineered) systems are structured (interdependently) and actually behave (i.e, exhibiting non-stationary, non-equilibrium and non-separable state dependence).

- Static Theoretical Ideal (classical linear framework)
- equilibrium
- stable
- separable (decomposable, study piecewise)
- Granger
- classic parametric models

- Dynamic Reality (nonlinear empirical dynamics)
 - non-equilibrium
 - non-stable
 - non-separable (interdependent, study as a whole)
- CCM
 - empirical dynamic models

The basic dichotomy here is a contrast between what was thought to be a necessary expedient (a theoretical compromise based on a static equilibrium system of independent parts), and the reality of nonlinear interdependent ever-changing natural systems.

The explosion of data is enabling investigation at the whole systems level.

What I tried to suggest today is that it is possible and worthwhile to develop approaches where this expedient is NOT necessary.

click



This Commentary appeared in PNAS a year ago and was a nice confirmation of the idea.

-data science, makes this all possible...data driven discovery